MODELOS MATEMATICOS

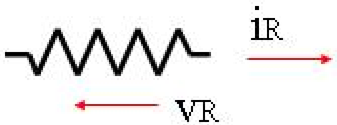
⇨ **HIPOTESIS SIMPLIFICATIVAS:**

- EFECTOS PEQUEÑOS
- PARAMETROS CONCENTRADOS
- LINEALIDAD
- PARAMETROS CONSTANTES

⇨ **SISTEMAS FISICOS**

- ELECTRICOS
- MECANICOS TRASLACIONALES
- MECANICOS ROTACIONALES
- HIDRAULICOS
- NEUMATICOS
- TERMICOS

SISTEMAS ELECTRICOS

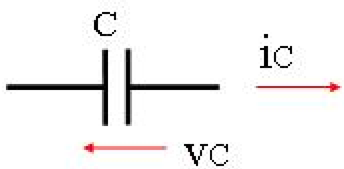


RESISTENCIA

$$i_R(t) = \frac{1}{R} v_R(t) \xrightarrow{L} I_R(s) = \frac{1}{R} V_R(s)$$

INDUCTANCIA

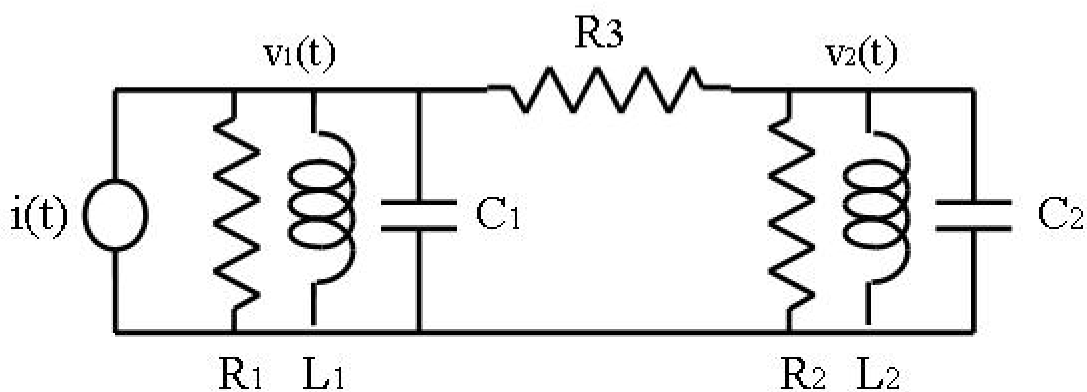
$$i_L(t) = \frac{1}{L} \int v_L(t) dt \xrightarrow{L} I_L(s) = \frac{1}{sL} V_L(s)$$



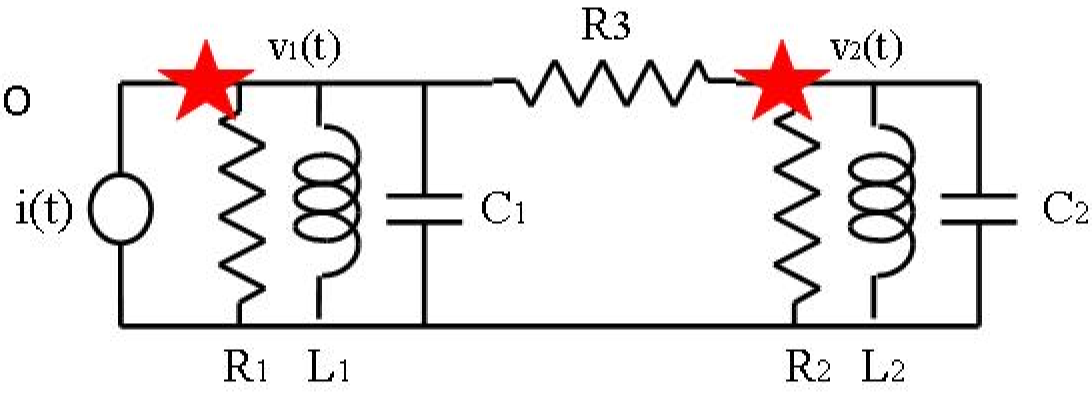
CAPACITANCIA

$$i_C(t) = C \frac{dv_C(t)}{dt} \xrightarrow{L} I_C(s) = sCV_C(s)$$

⇨ EJEMPLO: Encontrar la Transferencia $\frac{V_2(s)}{I_1(s)}$



Sistemas Eléctricos - Ejemplo



Nodo 1

$$i_1(t) = C_1 \frac{dv_1(t)}{dt} + \frac{v_1(t)}{R_1} + \frac{v_1(t)}{R_3} + \frac{1}{L_1}\int v_1(t)dt - \frac{v_2(t)}{R_3}$$

$$\therefore \rightarrow \; I_1(s) = (C_1 s + \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{sL_1})V_1(s) - \frac{1}{R_3}V_2(s)$$

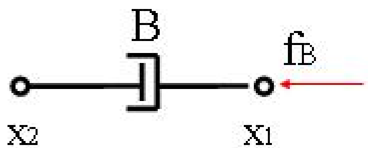
Nodo 2

$$0 = -\frac{v_1(t)}{R_3} + C_2 \frac{dv_2(t)}{dt} + \frac{v_2(t)}{R_2} + \frac{v_2(t)}{R_3} + \frac{1}{L_2}\int v_2(t)dt$$

$$\therefore \rightarrow \; 0 = -\frac{1}{R_3}V_1(s) + (C_2 s + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{sL_2})V_2(s)$$

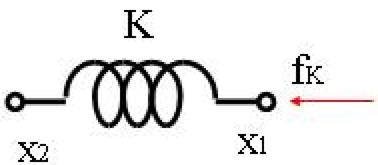
$$\frac{V_2(s)}{I_1(s)}$$

SISTEMAS MECANICOS TRASLACIONALES



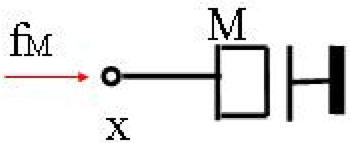
FRICCION

$$f_B(t)=B\left(\frac{dx_1(t)}{dt}-\frac{dx_2(t)}{dt}\right)\xrightarrow{L}F_B(s)=sB\left(X_1(s)-X_2(s)\right)$$



ELASTANCIA

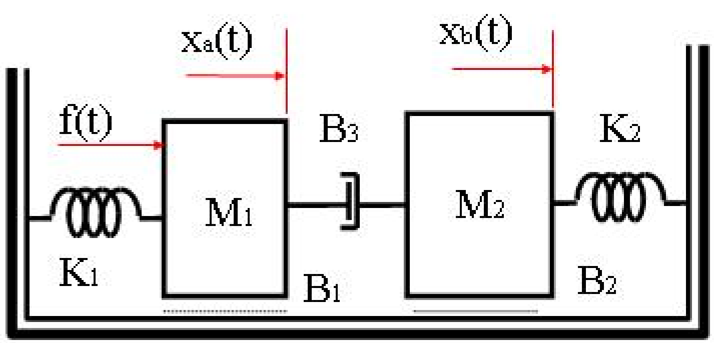
$$f_K(t)=K(x_1(t)-x_2(t))\xrightarrow{L}F_K(s)=K(X_1(s)-X_2(s))$$



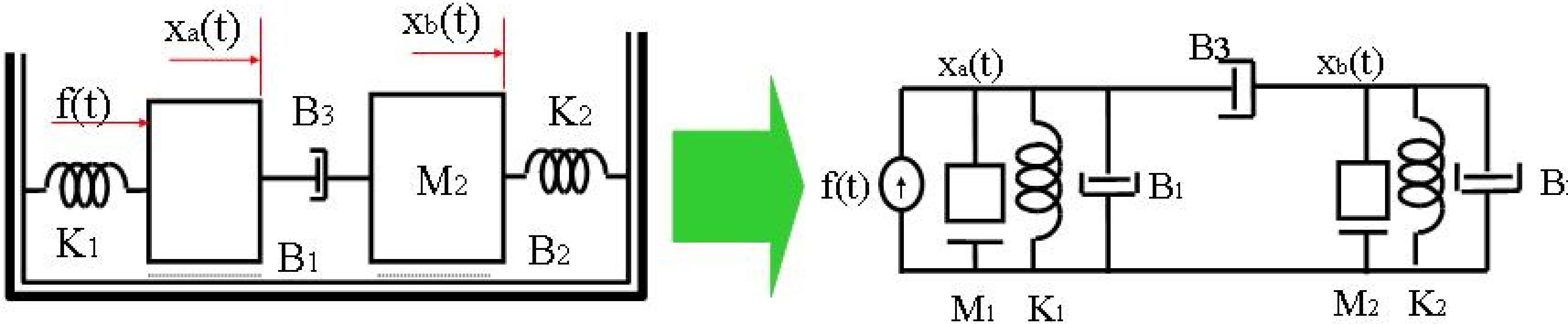
MASA INERCIAL

$$f_M(t)=M\frac{d^2x(t)}{dt^2}\xrightarrow{L}F_M(s)=s^2MX(s)$$

⇨ EJEMPLO: Encontrar la transferencia $\frac{X_a(s)}{F(s)}$



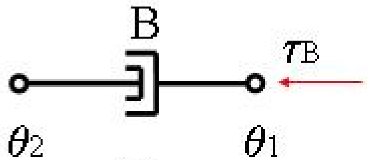
Sistemas Mecánicos Traslacionales - Ejemplo



$$
\begin{cases}
NODO.a \rightarrow F(s) = (M_1 s^2 + B_1 s + B_3 s + K_1) X_a(s) - B_3 s X_b(s) \\
NODO.b \rightarrow 0 = -B_3 s X_a(s) + (M_2 s^2 + B_2 s + B_3 s + K_2) X_b(s)
\end{cases}
$$

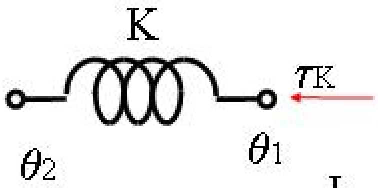
$$
\frac{X_a(s)}{F(s)}
$$

SISTEMAS MECANICOS ROTACIONALES



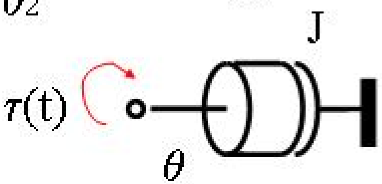
FRICCION

$$\tau_B(t) = B\left(\frac{d\theta_1(t)}{dt} - \frac{d\theta_2(t)}{dt}\right) \xrightarrow{L} T_B(s) = Bs(\Theta_1(s) - \Theta_2(s))$$



ELASTANCIA

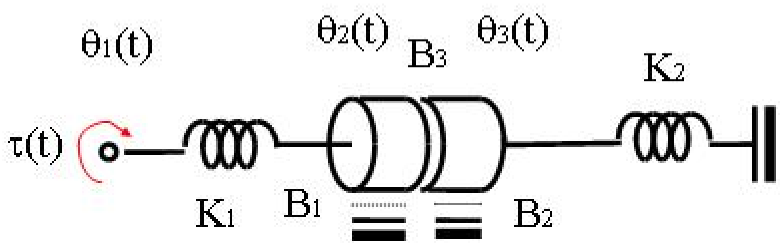
$$\tau_K(t) = K(\theta_2(t) - \theta_1(t)) \xrightarrow{L} T_K(s) = K(\Theta_1(s) - \Theta_2(s))$$



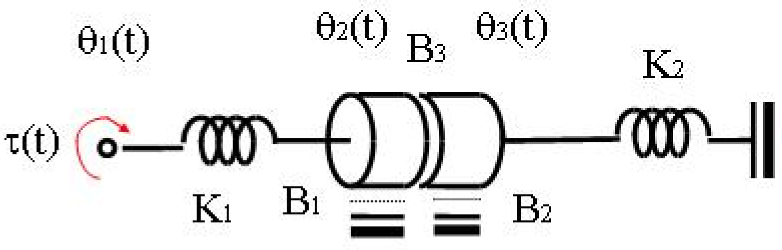
MOMENTO INERCIAL

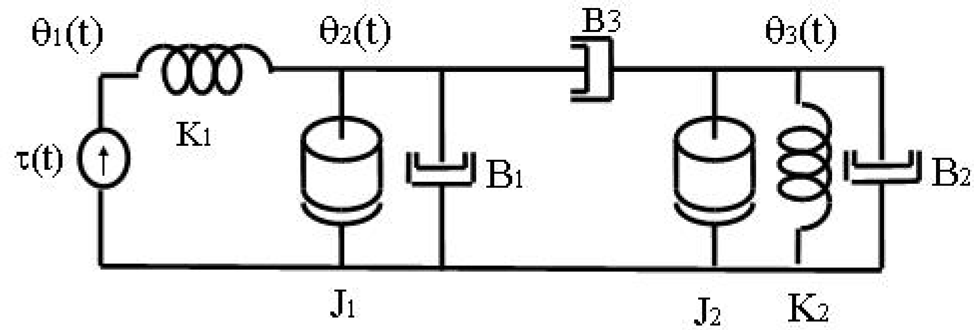
$$\tau_M(t) = J\frac{d^2\theta(t)}{dt^2} \xrightarrow{L} T_M(s) = s^2 J\Theta(s)$$

⇨ EJEMPLO: Encontrar la transferencia $\dfrac{\theta_2(s)}{T(s)}$



Sistemas Mecánicos Rotacionales - Ejemplo

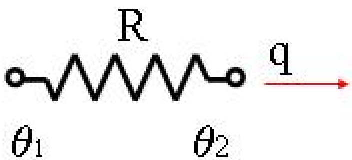




$$
\begin{cases}
NODO{:}\theta_1 \rightarrow \mathrm{T}(s) = K_1\theta_1(s) - K_1\theta_2(s) \\
NODO{.}\theta_2 \rightarrow 0 = -K_1\theta_1(s) + (J_1s^2 + B_1s + B_3s + K_1)\theta_2(s) - B_3s\theta_3(s) \\
NODO{.}\theta_3 \rightarrow 0 = -B_3s\theta_2(s) + (J_2s^2 + B_2s + B_3s + K_2)\theta_3(s)
\end{cases}
$$

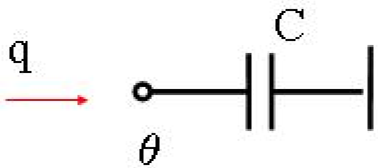
$$
\frac{\Theta_2(s)}{\mathrm{T}(s)}
$$

SISTEMAS TERMICOS



RESISTENCIA

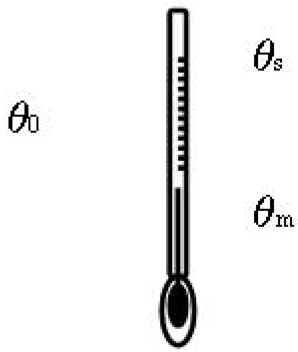
$$q(t)=\frac{1}{R}\left(\theta_1(t)-\theta_2(t)\right)\xrightarrow{L}Q(s)=\frac{1}{R}\left(\Theta_1(s)-\Theta_2(s)\right)$$



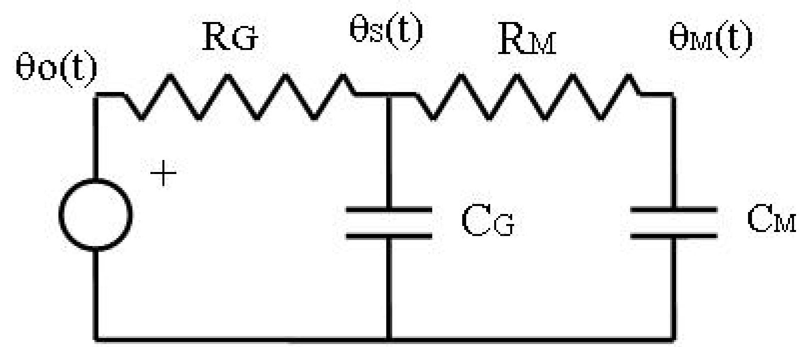
CAPACIDAD

$$q(t)=C\frac{d\theta(t)}{dt}\xrightarrow{L}Q(s)=sC\Theta(s)$$

EJEMPLO: Termómetro Clínico, encontrar: $\frac{\Theta_M(s)}{\Theta_0(s)}$

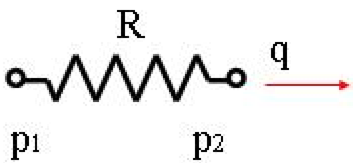


Rg=R vidrio-aire
Rm=R mercurio-vidrio
CG=C vidrio
CM=C mercurio



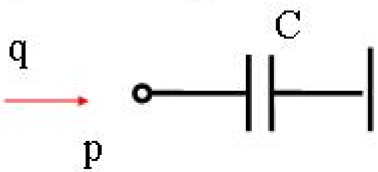
$$\frac{\Theta_M(s)}{\Theta_0(s)} \Leftarrow \begin{cases} NODO:S \rightarrow 0 = -\frac{\Theta_0(s)}{R_G}-\frac{\Theta_M(s)}{R_M}+\left(\frac{1}{R_G}+\frac{1}{R_M}+sC_G\right)\Theta_S(s) \\ NODO:M \rightarrow 0 = -\frac{\Theta_S(s)}{R_M}+\left(\frac{1}{R_M}+sC_M\right)\Theta_M(s) \end{cases}$$

SISTEMAS HIDRAULICOS



RESTRICCION

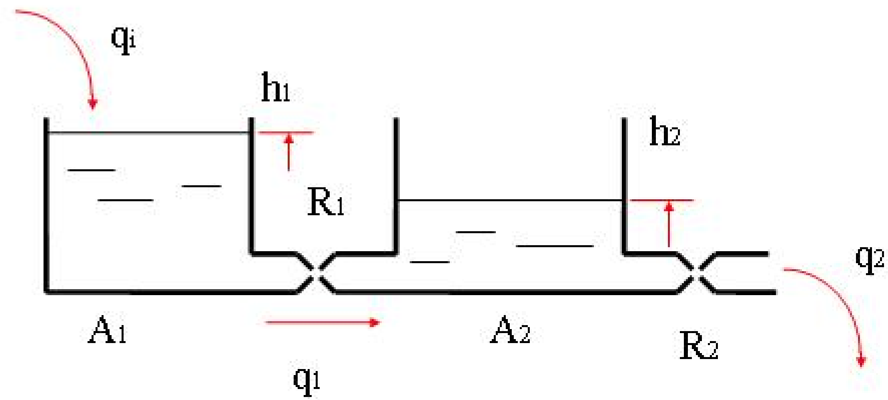
$$q(t)=\frac{1}{R}\left(p_1(t)-p_2(t)\right)\xrightarrow{L}Q(s)=\frac{1}{R}\left(P_1(s)-P_2(s)\right)$$

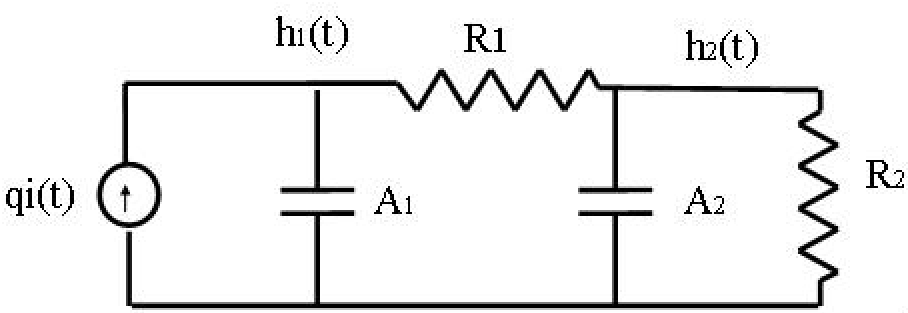


CAPACIDAD

$$q(t)=C\,\frac{dp(t)}{dt}\xrightarrow{L}Q(s)=sCP(s)$$

EJEMPLO: Hallar la transferencia: $\dfrac{H_1(s)}{Q_1(s)}$



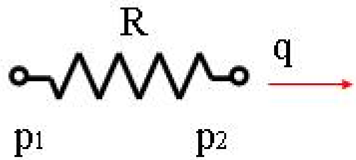


$$NODO{:}1\rightarrow Q_i(s)=\left(\frac{1}{R_1}+sA_1\right)H_1(s)-\frac{H_2(s)}{R_1}$$

$$NODO{:}2\rightarrow 0=-\frac{H_1(s)}{R_1}+\left(\frac{1}{R_1}+\frac{1}{R_2}+sA_2\right)H_2(s)$$

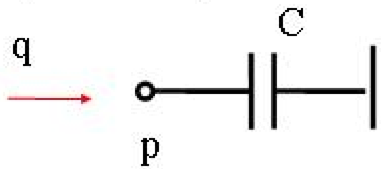
$$\frac{H_1(s)}{Q_1(s)}$$

SISTEMAS NEUMATICOS



RESTRICCION

$$q(t) = \frac{1}{R}\left(p_1(t) - p_2(t)\right) \xrightarrow{L} Q(s) = \frac{1}{R}\left(P_1(s) - P_2(s)\right)$$



CAPACIDAD

$$q(t) = C\frac{dp(t)}{dt} \xrightarrow{L} Q(s) = sCP(s)$$

A Volumen y Temperatura Constantes:

$$\frac{dm}{dp} = \frac{d(V\delta)}{dp} = V\frac{d\delta}{dp}$$

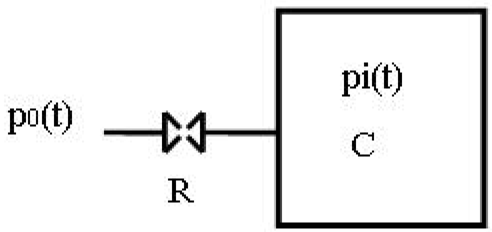
$$pv = RT \rightarrow \frac{p}{\delta} = RT \rightarrow \frac{d\delta}{dp} = \frac{1}{RT}$$

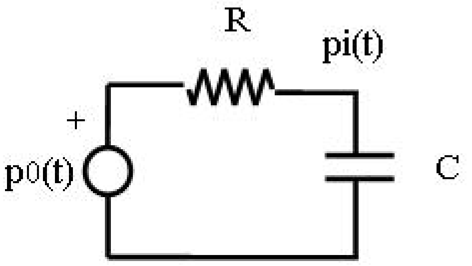
$$\therefore C = \frac{V}{RT}$$

$$\left.\begin{aligned} C &= \frac{dm}{dp} \\ q(t) &= \frac{dm}{dt} \end{aligned}\right\} \therefore q(t) = C\frac{dp}{dt}$$

SISTEMAS NEUMATICOS - Ejemplo

Hallar la Transferencia: $\dfrac{P_1(s)}{P_0(s)}$

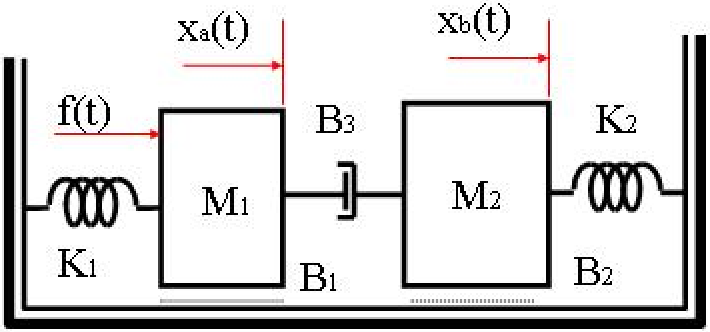


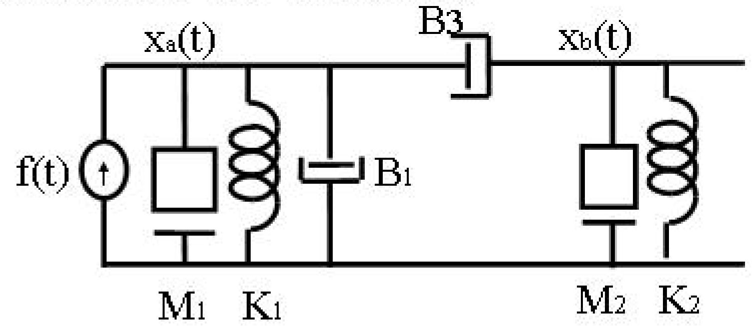


En el nodo p_1:

$$\begin{cases} 0 = -Rp_0(t) + Rp_1(t) + C\hat{p}_1(t) \therefore \\ 0 = -RP_0(s) + RP_1(s) + sCP_1(s) \therefore \\ RP_0(s) = P_1(s)\left(R + sC\right) \therefore \\ \dfrac{P_1(s)}{P_0(s)} = \dfrac{R}{R + sC} = \dfrac{1}{1 + sCR} \end{cases}$$

TRANSFERENCIA – MODELO DE ESTADO





Obtención de la Transferencia:

$$\begin{cases} f(t) = M_1\ddot{x}_a + (B_1 + B_3)\dot{x}_a + K_1 x_a - B_3\dot{x}_b \\ 0 = -B_3\dot{x}_a + M_2\ddot{x}_b + (B_2 + B_3)\dot{x}_b + K_1 x_b \end{cases}$$

transformando:

$$\begin{cases} F(s) = \left(M_1 s^2 + (B_1 + B_3)s + K_1\right)X_a(s) - B_3 s X_b(s) \\ 0 = -B_3 s X_a(s) + \left(M_2 s^2 + (B_2 + B_3)s + K_2\right)X_b(s) \end{cases}$$

luego, reemplazando:

$$F(s) = \left(M_1 s^2 + (B_1 + B_3)s + K_1\right)X_a(s) - \frac{B_3^2 s^2 X_a(s)}{\left(M_2 s^2 + (B_2 + B_3)s + K_2\right)}$$

$$\therefore \frac{X_a(s)}{F(s)} = \frac{\left(M_2 s^2 + (B_2 + B_3)s + K_2\right)}{\left(M_1 s^2 + (B_1 + B_3)s + K_1\right)\left(M_2 s^2 + (B_2 + B_3)s + K_2\right) - s^2 B_3^2}$$

f(t)

$$\frac{\left(M_2 s^2 + (B_2 + B_3)s + K_2\right)}{M_1 M_2 s^4 + (B_1 + B_2 + B_3)s^3 + \left\{M_1 K_2 + M_2 K_1 + \left[(B_1 + B_3)(B_2 + B_3) - B_3^2\right]\right\}s^2 + \left[(B_1 + B_3)K_2 + (B_2 + B_3)K_1\right]s + K_1 K_2}$$

x(t)

MODELO DE ESTADO

$$\begin{cases} f(t) = M_1\ddot{x}_a + (B_1 + B_3)\dot{x}_a + K_1 x_a - B_3\dot{x}_b \\ 0 = -B_3\dot{x}_a + M_2\ddot{x}_b + (B_2 + B_3)\dot{x}_b + K_1 x_b \end{cases}$$

asignación :

$$x_1 = x_a ; x_2 = x_b ; x_3 = \dot{x}_a ; x_4 = \dot{x}_b ; u = f$$

$$u(t) = M_1\dot{x}_3 + (B_1 + B_3)x_3 + K_1 x_1 - B_3 x_4$$

$$0 = -B_3 x_3 + M_2\dot{x}_4 + (B_2 + B_3)x_4 + K_1 x_2$$

MODELO DE ESTADO

$$\begin{cases} \dot{x}_1 = x_3 \\ \dot{x}_2 = x_4 \\ \dot{x}_3 = -\dfrac{K_1}{M_1}x_1 + 0x_2 - \dfrac{(B_1+B_3)}{M_1}x_3 + \dfrac{B_3}{M_1}x_4 + \dfrac{1}{M_1}u \\ \dot{x}_4 = 0x_1 - \dfrac{K_2}{M_2}x_2 + \dfrac{B_3}{M_2}x_3 - \dfrac{(B_2+B_3)}{M_2}x_4 \end{cases}$$

$$\dot{\overline{x}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{K_1}{M_1} & 0 & -\dfrac{(B_1+B_3)}{M_1} & \dfrac{B_3}{M_1} \\ 0 & -\dfrac{K_2}{M_2} & \dfrac{B_3}{M_2} & \dfrac{(B_2+B_3)}{M_2} \end{bmatrix}\overline{x} + \begin{bmatrix} 0 \\ 0 \\ -\dfrac{1}{M_1} \\ 0 \end{bmatrix}u$$

$$c = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}\overline{x}$$